Summer Work

SUPA Calculus 3

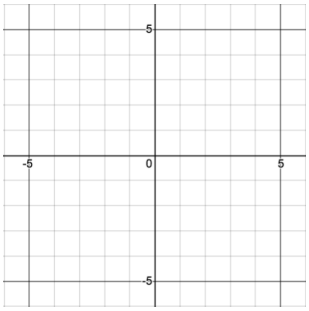
Please complete all problems and be ready to turn it in at the beginning of the school year. You can use any resources online for reference, graphing, or calculations, but please show any necessary work. A useful graphing tool is `desmos.com/calculator`.

You can email Mr. Caulfield at `mcaulfield@rih.org` with any questions.

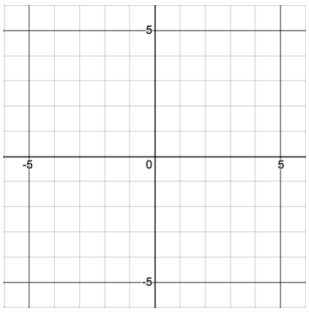
Conic sections

For each of the following, graph the conic section and identify the type.

1. $\frac{x^2}{9} + \frac{y^2}{16} = 1$

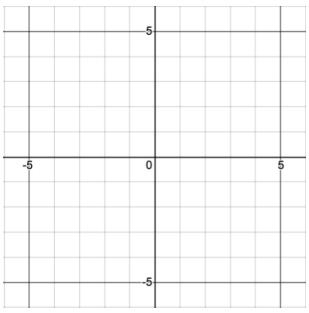


2. $\frac{x^2}{4} - y^2 = 1$



Parametric equations

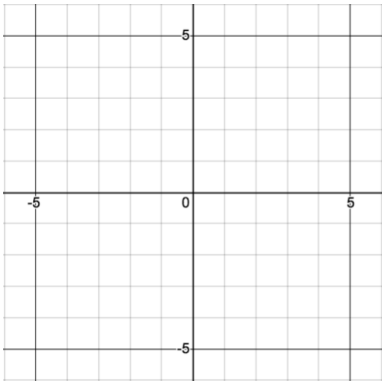
3. Sketch the curve. Indicate the direction of increasing t with an arrow.
   $x = 1 + \sqrt{t}, y = t^2 - 4t$ where $0 \leq t \leq 4$.



4. For the curve:

   $x = 5\cos t, y = 3\sin t$ where $0 \leq t \leq 2\pi$.

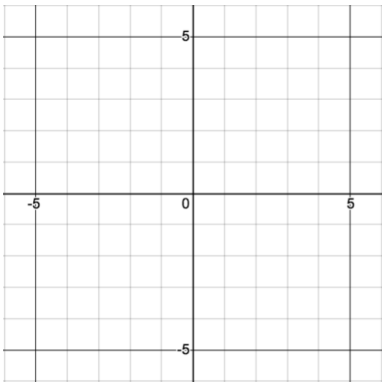
   Sketch the curve. Indicate the direction of increasing t with an arrow. Then find, in point-slope form, the equation for the tangent line to the curve at $t = \pi/4$, and graph the tangent line.



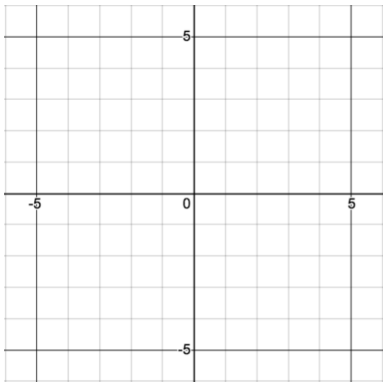
Polar coordinates

Sketch each inequality as a region in the plane.

5. $1 \leq r \leq 3$



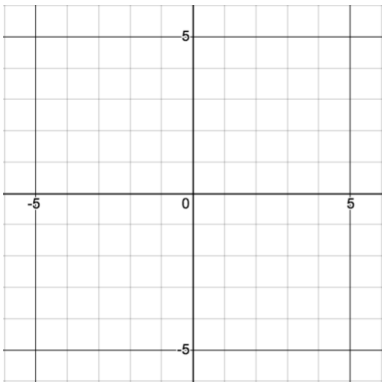
6. $\dfrac{\pi}{4} \leq \theta \leq \dfrac{\pi}{3}$



7. Sketch the curve:

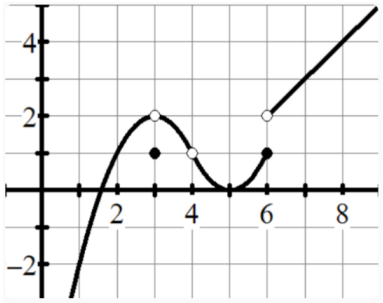
   $r = 4\sin\theta$

   $0 \leq \theta \leq \pi$



Limits

8. Consider the graph of the function $f(x)$ below:



Find each of the following, or determine if it's undefined.

(a) $f(3)$

(b) $f(4)$

(c) $f(6)$

(d) $\lim_{x\to 3} f(x)$

(e) $\lim_{x\to 4} f(x)$

(f) $\lim_{x\to 6} f(x)$

(g) $\lim_{x\to 6^+} f(x)$

(h) $\lim_{x\to 6^-} f(x)$

9. Find each limit or determine that the limit does not exist.

(a) $\lim_{x\to 0} \dfrac{(x+1)^2 - 1}{x}$

(b) $\lim_{x\to 0} \dfrac{2|x|}{3x}$

Derivatives

10. Find, in slope-intercept form, the equation for the tangent line to $f(x) = x^3 + 2x^2 - 4$ at $x = -1$.

11. Find the derivative of the function $f(x) = e^{2x}x^{10}$

12. Find the derivative of the function, $f(x) = \dfrac{x^2 + 5x}{2x + 1}$

13. Starting with the derivatives of sine and cosine, use the Quotient Rule and any necessary trig identities to show that,

$$\frac{d}{dx}\cot x = -\csc^2 x$$

14. Find and classify all critical points of the function:
    $f(x) = x^3 - 12x + 1$

15. Find the derivative of the function, $f(x) = \sin(\ln(2x))$.

Integrals

Evaluate each of the following:

16. $\int_0^1 x^2(1+2x^3)^5 dx$
    Hint, use U-substitution.

17. Consider the integral $\int_0^7 f(x)dx$, where:

$$f(x) = \begin{cases} 3x & 0 \leq x \leq 2 \\ 6 & 2 \leq x \leq 7 \end{cases}$$

    Sketch the function on the interval $0 \leq x \leq 7$ and shade an area representing the intergral. Then use geometry not calculus to evaluate the integral.

18. Evaluate $\int_1^2 x^4 \log_2(x) dx$
    Hint: Use change of base and integration by parts.

19. Find the area enclosed by the two curves, $y = x^2 - 4x + 6$ and $y = x + 2$.

20. Find the average value of $f(x) = x \sin x$ on the interval $0 \leq x \leq \pi$. The average value is the integral, divided by the length of the interval. Again, use integration by parts.